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# PERIODIC PULSE HEATING OF METALS

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The article deals with the effect of the time-dependent form and heating by preceding pulses on the resulting temperature field in metal subjected to periodic pulsed heat treatment.

The widespread use of lasers in various technological processes requires the construction of theoretical models describing the effect of laser radiation on substances. At present there are available many theoretical investigations dealing with continuous and pulsed treatment of materials; their results were generalized, e.g., in [1-7]. However, periodic pulsed loading received much less attention [8, 9] although in practice it is ever more widely used.

A unidimensional temperature field T(x, t) in a half-space with an arbitrary time dependence of the energy flux density can be written in the form [10]

$$T = T_n + \frac{1}{\lambda}\sqrt{\frac{a}{\pi}}\int_0^t q(\xi)\,\frac{\exp[-x^2/4a(t-\xi)]}{\sqrt{t-\xi}}\,d\xi. \tag{1}$$

For expression (1) we use the Laplace–Carson transformation [11]

$$\bar{T} = T_n + \frac{\sqrt{a}}{\lambda}\,\frac{\bar{q}(p)}{\sqrt{p}}\exp(-x\sqrt{p/a}). \tag{2}$$

To find $\bar{q}(p)$, we expand q(t) into a Fourier series, Eq. (2) assumes the form

$$\bar{T} = T_n + \frac{\sqrt{a}}{\lambda}\sum_{k=-\infty}^{+\infty} c_k\,\frac{\sqrt{p}\exp(-x\sqrt{p/a})}{p - i\omega_k},$$

where

$$c_k = \frac{1}{\tau}\int_0^\tau q(t)\exp(-i\omega_k t)\,dt,\ \ \omega_k = k\omega_0,\ \ k = 0,\ 1,\ \ldots,$$

$\omega_0 = 2\pi/\tau$, $\tau$ is the period of action, i.e., the interval between the instants of the onset of two adjacent pulses. Going over to the original, we have

$$T = T_n + \frac{\sqrt{a}}{\lambda}\sum_{k=-\infty}^{+\infty} c_k\,\frac{(i-1)\exp(i\omega_k t)}{2\sqrt{2\omega_k}}\left\{\exp\left[\frac{x}{2}\sqrt{\frac{2\omega_k}{a}}(i+1)\right]\times\right.$$

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$$\times \operatorname{erfc}\left[\frac{x}{2\sqrt{at}}+\sqrt{\frac{\omega_k t}{2}(i+1)}\right]-\exp\left[-\frac{x}{a}\sqrt{\frac{2\omega_k}{a}(i+1)}\right]\operatorname{erfc}\left[\frac{x}{2\sqrt{at}}-\sqrt{\frac{\omega_k t}{2}(i+1)}\right]\Bigg\}. \quad (3)$$

The obtained expression gives the temperature field in metal with an arbitrary time-dependent pulse shape. The pulse, normalized at the maximum to the unit $\tilde{q} = q/q_0$ and described as the function $\xi = t/\tau_0$, has practically the same shape for many types of laser if we neglect the high-frequency structure and regard only the envelope [3] which, with good accuracy, is approximated by the trapezoidal shape

$$\tilde{q}(\xi) = \begin{cases} \xi/\xi_1, & 0 \leqslant \xi \leqslant \xi_1,\ \xi_1 \simeq 0{,}15, \\ 1, & \xi_1 \leqslant \xi \leqslant \xi_2,\ \xi_2 = 0{,}21, \\ (\xi-1)/(\xi_2-1), & \xi_2 \leqslant \xi \leqslant \xi_3,\ \xi_3 = 1, \end{cases} \quad (4)$$

and expression (3) can therefore be considerably simplified. When we substitute (4) into Eq. (1), we have

$$\tilde{T} = \frac{1}{3}\sum_{k=0}^{n-1}\sum_{i=0}^{3}\frac{(-1)^i(\xi-\xi_i-k\eta)^{3/2}}{\xi_1(\delta_{0,i}+\delta_{1,i})-(1-\xi_2)(\delta_{2,i}+\delta_{3,i})}\times$$
$$\times\left\{\left(3+\frac{2z^2}{\xi-\xi_i-k\eta}\right)\sqrt{\pi}\,\operatorname{ierfc}\left(\frac{z}{\sqrt{\xi-\xi_i-k\eta}}\right)-\exp\left(-\frac{z^2}{\xi-\xi_i-k\eta}\right)\right\}. \quad (5)$$

Here, $\tilde{T} = T/T_0$, $T_0 = 2q_0\sqrt{a\tau_0/\pi}/\lambda$, $\eta = \tau/\tau_0$, $z = x/2\sqrt{a\tau_0}$. At the initial instant we put the temperature of the metal equal to zero, $T_n = 0$. Expression (5) describes the temperature field in the n-th load cycle $(n-1)\eta \leq \xi \leq n\eta$, where for $k = n - 1$, summing with respect to i includes only the terms for which $\xi_i + (n-1)\eta < \xi$. When in (5) we let $\xi_1$ tend to zero, and $\xi_2$ to unity, we obtain the temperature field for rectangular pulses

$$\tilde{T} = \sqrt{\pi}\sum_{k=0}^{n-1}\left\{\sqrt{\xi-k\eta}\,\operatorname{ierfc}\left(\frac{z}{\sqrt{\xi-k\eta}}\right)-\sqrt{\xi-1-k\eta}\,\operatorname{ierfc}\left(\frac{z}{\sqrt{\xi-1-k\eta}}\right)\right\}. \quad (6)$$

With n = 1, relations (5), (6) describe the change of temperature for single-pulse loading. The temperature of the surface z = 0 in accordance with (5), (6) is equal to:

$$\tilde{T}(0,\xi) = \frac{2}{3}\sum_{k=0}^{n-1}\sum_{i=0}^{3}\frac{(-1)^i(\xi-\xi_i-k\eta)^{3/2}}{\xi_1(\delta_{0,i}+\delta_{1,i})-(1-\xi_2)(\delta_{2,i}+\delta_{3,i})}, \quad (7)$$

$$\tilde{T}(0,\xi) = \sum_{k=0}^{n-1}\{\sqrt{\xi-k\eta}-\sqrt{\xi-1-k\eta}\}. \quad (8)$$

Since in modern lasers operating in periodic pulse regime the pumping time is much longer than the pulse [12], we write $\xi$ in the form $\xi = (n-1)\eta + \theta$, where $\theta$ is measured from the instant of the beginning of the n-th pulse, $0 \leq \theta \leq \eta$, expressions (7), (8) can be changed to the common form

$$\tilde{T}(0,\xi) = \tilde{T}_1(0,\theta) + \sum_{k=1}^{n-1}\frac{\alpha}{(\theta+k\eta)^{1/2}}. \quad (9)$$

Here, $\tilde{T}_1(0, \theta)$ is the change of the surface temperature induced by the first pulse, $\alpha = (1 + \xi_2 - \xi_1)/4$. The use of diaphragms, e.g., in the form of a rotating disk with sectors cut out for modulating the continuous radiation into a periodically pulsed one makes it possible to obtain various pulse shapes, and also to vary the pulse width and the interval between pulses. Figure 1 shows the change of the surface temperature of a molybdenum specimen according to the experimental data (solid curves) and according to the calculation by formula (9) (dashed curves). With constant flux densities the surface temperature is maximal at the instant the pulse ends (Fig. 1a) whereas with variable radiation intensity the surface temperature attains its maximum during the action of the pulse (Fig. 1b, c). Thus, at the instant of action of the first pulse the surface temperature is maximal at the instant

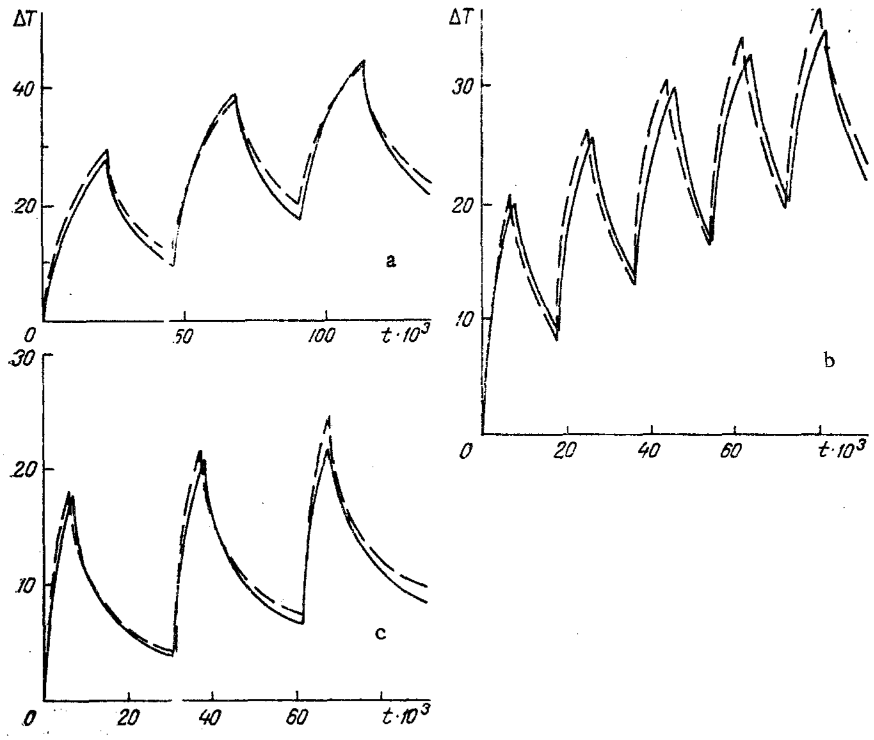


Fig. 1. Change of the surface temperature of a molybdenum specimen: a) rectangular pulse, $\tau_0 = 23 \cdot 10^{-3}$ sec, $q_0 = 3.5 \cdot 10^6$ W/m², $\eta = 2$; b, c) trapezoidal pulse, b) $\tau_0 = 9 \cdot 10^{-3}$ sec, $q_0 = 3.17 \cdot 10^6$ W/m², $\eta = 2$; c) $\tau_0 = 8 \cdot 10^{-3}$ sec, $q_0 = 2.77 \cdot 10^6$ W/m², $\eta = 4$. ΔT, °K; t, sec.

$$\xi_{m1} = \frac{\xi_1 + \xi_2(2-\omega_0)}{4-\omega_0}\left\{1 + \left[1 + \frac{\omega_1(4-\omega_0)}{(\xi_1+\xi_2(2-\omega_0))^2}\right]^{1/2}\right\},$$

$$\omega_0 = \xi_1^2/(1-\xi_2)^2, \quad \omega_1 = (1-\xi_2)^2 + \xi_2^2\omega_0 - 2\xi_1\xi_2.$$

By changing the number of pulses we can obtain the required temperature with fixed radiation intensity. The instant $\xi_{mn}$ of attaining the maximal temperature in the course of action of the n-th pulse can be written in the form $\xi_{mn} = \xi_{m1} + (n - 1)\eta$.

When metal surfaces are heat-treated by concentrated energy fluxes, it is indispensable to know the rates of change of temperature because when these rates are very high (~10⁸ °C/sec or more [1]), they lead to a shift of the critical points of phase transformations, to the appearance of various microstructures in the subsurface layer including metastable ones. Using expressions (5), (6), we find:

$$\frac{\partial \tilde{T}(z,\xi)}{\partial \xi} = \sum_{k=0}^{n-1}\sum_{i=0}^{3} \frac{\sqrt{\pi}(-1)^i(\xi-\xi_i-k\eta)^{1/2}\,\mathrm{ierfc}\,[z/(\xi-\xi_i-k\eta)^{1/2}]}{\xi_1(\delta_{0,i}+\delta_{1,i})-(1-\xi_2)(\delta_{2,i}+\delta_{3,i})}, \tag{10}$$

$$\frac{\partial \tilde{T}(z,\xi)}{\partial \xi} = \frac{1}{2}\sum_{k=0}^{n-1}\left\{\frac{\exp[-z^2/(\xi-k\eta)]}{(\xi-k\eta)^{1/2}} - \frac{\exp[-z^2/(\xi-1-k\eta)]}{(\xi-1-k\eta)^{1/2}}\right\}. \tag{11}$$

The instants $\xi_{mn}^h$ and $\xi_{mn}^c$ of attaining the maximal heating and cooling rates in the section z = 0 during the n-th load cycle are determined from the solution of the transcendental equation

$$\sum_{k=0}^{n-1}\sum_{i=0}^{3} \frac{(-1)^i \exp[-z^2/(\xi-\xi_i)]/(\xi-\xi_i)^{1/2}}{\xi_1(\delta_{0,i}+\delta_{1,i})-(1-\xi_2)(\delta_{2,i}+\delta_{3,f})} = 0 \tag{12}$$

for variable radiation intensity in a pulse and

$$\sum_{k=0}^{n-1}\left\{\frac{\exp[-z^2/(\xi-k\eta)]}{(\xi-k\eta)^{3/2}}\left(1-\frac{2z^2}{\xi-k\eta}\right)-\frac{\exp[-z^2/(\xi-1-k\eta)]}{(\xi-1-k\eta)^{3/2}}\left(1-\frac{2z^2}{\xi-1-k\eta}\right)\right\}=0 \qquad (13)$$

for constant flux density. In accordance with (9), the rate of change of the surface temperature $\gamma = \partial\tilde{T}(0, \xi)/\partial\xi$, is equal to

$$\gamma=\gamma_1-\sum_{k=1}^{n-1}\frac{\alpha}{2(\theta+k\eta)^{3/2}}, \qquad (14)$$

where $\gamma_1 = \partial\tilde{T}_1(0, \theta)/\partial\theta$. The heating rate of the surface $\gamma_1$ is maximal at the instant $\xi^h_{m1} = \xi_1$, i.e., in the case of a rectangular pulse at the instant of the onset of action. The cooling rate of the surface, however, has its highest value at the instant the pulse ends (Fig. 2). It follows from expression (14) that the heating and cooling rates during successive pulses ($n \geq 2$) are maximal at the instants $\xi^h_{mn} \approx \xi_1 + (n-1)\eta$ and $\xi^c_{mn} \approx 1 + (n-1)\eta$, respectively. The heating rates somewhat decrease and the cooling rates increase, but these changes are very small. For instance, the heating rate at the instant $\xi = 90 + \xi_1 (n = 10, \eta = 10)$ differs from its value for $\xi = \xi_1$ by a mere 0.3%, and the cooling rates at the instants of the end of the tenth and the first pulses differ by 1.2% from each other. Consequently, also the rate of change of the temperature for $z \neq 0$ in the course of periodic pulse treatment changes practically periodically with the period $\eta$. Thus, to determine the heating and cooling rates in the course of periodic pulsed loading it suffices to know the rate of change of the temperature caused by the first pulse (Fig. 2), the relations (10), (11) for $n = 1$. The instants of attaining the maximal heating and cooling rates at different depths are found from expressions (12), (13) for $n = 1$. The change of the maximal cooling rate vs. depth is presented in Fig. 3.

If during the process of heating the surface temperature remains sufficiently long in the range $\tilde{T}_f < \tilde{T}(0, \xi) < \tilde{T}_m$, then a phase transformation occurs which leads to a change of the surface properties of the material. In the course of periodic pulsed loading $\tilde{T}_f < \tilde{T}(0, \xi) < \tilde{T}_m$ with $n_0 < n < n_1$; $n_0$ and $n_1$ are determined from

$$\tilde{T}(0,\ (n_0-1)\eta)=\alpha\sum_{k=1}^{n_0-1}(k\eta)^{-1/2}>\tilde{T}_f, \qquad (15)$$

$$\tilde{T}(0,\ \xi_{mn})=\tilde{T}_1(0,\xi_{m1})+\alpha\sum_{k=1}^{n_1-1}(\xi_{m1}+k\eta)^{-1/2}<\tilde{T}_m. \qquad (16)$$

If we stipulate fixed values of $n_0$ and $n_1$, then the inequalities (15), (16) bound the permissible values of radiation intensity and of the period of action. After termination of the $n_1$-th pulse we carry out cooling up to the instant $\xi_s$, $\tilde{T}(0, \xi_s) = \tilde{T}_f$, and then we again switch on the radiating oscillator. The change of surface temperature for $\xi \geq \xi_s$ has the form

$$\tilde{T}(0,\ \xi)=\tilde{T}_1(0,\ \theta)+\alpha\left\{\sum_{k=0}^{n_1-1}(\psi+k\eta)^{-1/2}+\sum_{k=1}^{n-1}(\theta+k\eta)^{-1/2}\right\}.$$

Here, $\psi$ is measured from the onset of action of the $n_1$-th pulse, $\psi = (n-1)\eta + \theta + 1 + \psi_c$, $\psi_c$ is the duration of the cooling process from the end of the $n_1$-th pulse to the onset of the second series of pulses, $\theta$ is measured from the instant of the onset of action of the n-th pulse in the second series, $0 \leq \theta \leq \eta$. By arranging heat cycling in such a manner, it is possible to maintain the subsurface layer of metal for the required time in the specified temperature regime.

As an example we will examine the optimization of the radiation parameters in two-pulse loading. For the sake of simplicity we take the rectangular pulse shape. Assume that it is required that at the time of action of the second pulse $\tilde{T}_f < \tilde{T}(0, \xi) < \tilde{T}_m$. According to (6), at the instant of the onset $\xi = \eta$ and of the end $\xi = \eta + 1$ of the second pulse the following inequalities have to be fulfilled:

$$\sqrt{\eta}-\sqrt{\eta-1}>A_f/\varkappa_0,\quad \sqrt{\eta+1}-\sqrt{\eta}+1<A_m/\varkappa_0. \qquad (17)$$

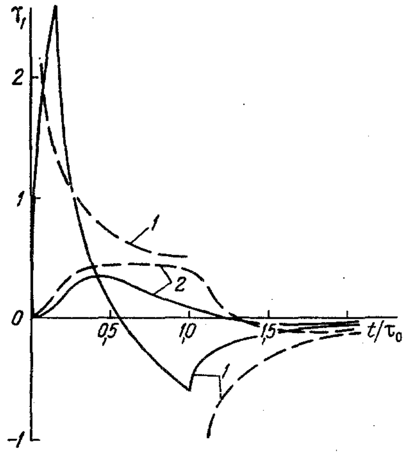


Fig. 2

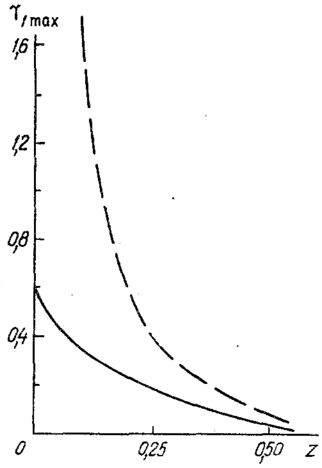


Fig. 3

Fig. 2. Rate of change of temperature with constant (dashed curves) and variable ($\xi_1$ = 0.15; $\xi_2$ = 0.21) (solid curves) energy flux density: 1) z = 0; 2) z = 0.5.

Fig. 3. Change of the maximal cooling rate vs depth with constant (dashed curve) and variable ($\xi_1$ = 0.15; $\xi_2$ = 0.21) (solid curve) energy flux density.

Here, $A_f = 0.5T_f\lambda\sqrt{\pi/a}$, $A_m = 0.5T_m\lambda\sqrt{\pi/a}$, $k_0 = q_0\sqrt{\tau_0}$. When $\eta >> 1$, (17) assumes the form

$$\lambda T_f\sqrt{\pi\eta/a} \leqslant \varkappa_0 \leqslant 0{,}5\lambda(T_m - T_f)\sqrt{\pi/a}.$$

The second inequality serves for determining the energy characteristics of a separate pulse, and the first inequality, with selected magnitude of $k_0(q_0$ and $\tau_0)$, yields the interval separating the onsets of the first and second pulses from each other

## NOTATION

T, temperature; $T_n$, initial temperature; $T_0$, surface temperature at the instant of the end of the rectangular pulse with power density $q_0$; $q_0$, energy flux density at the maximum; q, energy flux density at the instant t; x, three-dimensional coordinate; $\tau_0$, pulse width; $\tau$, period of action; $a$, thermal diffusivity of the metal; $\lambda$, thermal conductivity of the metal; $\delta_{ji}$, Kronecker delta; $\theta = \xi - (n - 1)\eta$; n, number of the given load pulse; $\gamma$, rate of change of the surface temperature in dimensionless form; $\tilde{T}_f = T_f/T_0$; $T_f$, temperature of phase transition in the solid state; $\tilde{T}_m = T_m/T_0$; $T_m$, melting point.

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